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Macao University of Tourism

澳門科技大學
UNIVERSIDADE DE CIÊNCIA E TECNOLOGIA DE MACAU
MACAU UNIVERSITY OF SCIENCE AND TECHNOLOGY

澳門四高校聯合入學考試（語言科及數學科）
Joint Admission Examination for Macao Four Higher Education Institutions (Languages and Mathematics)

2026 年試題及參考答案
2026 Examination Paper and Suggested Answer

數學附加卷 Mathematics Supplementary Paper

注意事項：

1. 考生獲發文件如下:

   1.1 本考卷包括封面共 22 版

   1.2 草稿紙一張

2. 請於本考卷封面填寫聯考編號、考場、樓宇、考室及座號。

3. 本考卷共有五條解答題，每題二十分，任擇三題作答。全卷滿分為六十分。

4. 必須在考卷內提供的橫間頁內作答，寫在其他地方的答案將不會獲評分。

5. 必須將解題步驟清楚寫出。只當答案和所有步驟正確而清楚地表示出來，考生方可獲得滿分。

6. 本考卷的圖形並非按比例繪畫。

7. 考試中不可使用任何形式的計算機。

8. 請用藍色或黑色原子筆作答。

9. 考試完畢，考生須交回本考卷及草稿紙。

Instructions:

1. Each candidate is provided with the following documents:

   1.1 Question paper including cover page – 22 pages

   1.2 One sheet of draft paper

2. Fill in your JAE No., campus, building, room and seat no. on the front page of the examination paper.

3. There are 5 questions in this paper, each carries 20 marks. Answer any 3 questions. Full mark of this paper is 60.

4. Put your answers in the lined pages provided. Answers put elsewhere will not be marked.

5. Show all your steps in getting to the answer. Full credits will be given only if the answer and all the steps are correct and clearly shown.

6. The diagrams in this examination paper are not drawn to scale.

7. Calculators of any kind are not allowed in the examination.

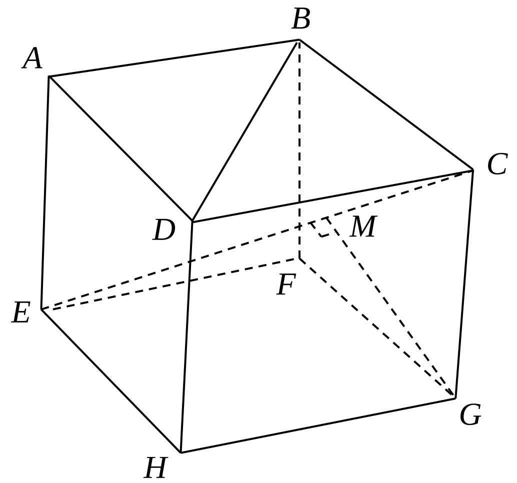
8. Answer the questions with a blue or black ball pen.

9. Candidates must return the question paper and draft paper at the end of the examination.

任擇三題作答，每題二十分。請把答案寫在緊接每條題目之後的 3 頁橫間頁內。

Answer any 3 questions, each carries 20 marks. Write down the answers on the 3 lined pages following each question.

1.



如上圖，$ABCD-EFGH$ 是正立方體，其邊長為 1。設 M 是 EC 上一點，使得 $GM \perp EC$ 。

(a) 證明 ΔCMG、ΔCMB 和 ΔCMD 是全等三角形。 (5 分)

(b) 證明 CM 垂直於平面 BDG，且 M 是平面 BDG 上一點。 (5 分)

(c) 一圓錐有 E 為頂點，其底圓通過 B、 D 和 G。求此圓錐的體積。 (10 分)

In the above figure, $ABCD-EFGH$ is a cube with edge length 1. Let M be the point on EC such that $GM \perp EC$.

(a) Show that ΔCMG, ΔCMB and ΔCMD are congruent triangles. (5 marks)

(b) Show that CM is perpendicular to the plane BDG, and M is a point in the plane BDG. (5 marks)

(c) A circular cone has E as the vertex and its base circle passes through B, D and G. Find the volume of the cone. (10 marks)

2. 已知函數 $f(x) = x^3 - x^2 - x$。

(a) (i) 求 $f'(x)$ 及 $f''(x)$。 (2 分)

(ii) 求 $f(x)$ 的局部極大值和局部極小值。 (4 分)

(iii) 求曲線 $y = f(x)$ 的拐點。 (2 分)

(iv) 繪出曲線 $y = f(x)$，$-2 \leq x \leq 2$。 (3 分)

(v) 繪出曲線$y = -f(|x|)$，$-2 \leq x \leq 2$。 (1 分)

(b) 求由曲線 $y = f(x)$ 及直線 $y = x$ 所包圍的兩區域的總面積。 (8 分)

Given the function $f(x) = x^3 - x^2 - x$.

(a) (i) Find $f'(x)$ and $f''(x)$. (2 marks)

(ii) Find the local maximum and local minimum values of $f(x)$. (4 marks)

(iii) Find the inflection point(s) of the curve $y = f(x)$. (2 marks)

(iv) Sketch the curve $y = f(x)$, $-2 \leq x \leq 2$. (3 marks)

(v) Sketch the curve $y = -f(|x|)$, $-2 \leq x \leq 2$. (1 mark)

(b) Find the total area of the two regions bounded by the curve $y = f(x)$ and the line $y = x$. (8 marks)

3. 已知橢圓 $E: \frac{x^2}{9} + \frac{y^2}{4} = 1$，其焦點為 $F_1(-a, 0)$ 及 $F_2(a, 0)$，當中 $a > 0$。設 C 是以 F_1F_2為直徑的圓。設 E 與 C 在第一象限的交點為 Q。

(a) 求 a 的值，並求 Q 的座標。 (5 分)

(b) 設直線 QF_2 與橢圓 E 的另一交點為 M ($M \neq Q$)，求 M 的座標。 (7 分)

(c) 求 $\tan\angle QMF_1$。 (4 分)

(d) 求 Q 與 直線 MF_1 的距離。 (4 分)

Given an ellipse $E: \frac{x^2}{9} + \frac{y^2}{4} = 1$ with foci $F_1(-a, 0)$ and $F_2(a, 0)$ where $a > 0$. Let C be the circle with F_1F_2 as a diameter. Suppose the intersection point of E and C in the first quadrant is Q.

(a) Find the value of a and the co-ordinates of Q. (5 marks)

(b) Suppose the line QF_2 and E intersect at another point M ($M \neq Q$). Find the co-ordinates of M. (7 marks)

(c) Find $\tan\angle QMF_1$. (4 marks)

(d) Find the distance between Q and line MF_1. (4 marks)

4. 設 $i=\sqrt{-1}$。

(a) (i) 求方程 $z^2-\sqrt{3}z+1=0$ 的兩個複數解並以極式 $\cos\theta+i\sin\theta$ 表示，其中 $-\pi<\theta\leq\pi$。 (5 分)

(ii) 用 (i) 的結果，求方程 $z^4-\sqrt{3}z^2+1=0$ 的所有複數解並以極式 $\cos\theta+i\sin\theta$ 表示，其中 $-\pi<\theta\leq\pi$。 (5 分)

(b) (i) 證明多項式 $z^4-\sqrt{3}z^2+1$ 有以下的因式分解:

$$z^4-\sqrt{3}z^2+1=(z^2-2z\cos\frac{\pi}{12}+1)(z^2+2z\cos\frac{\pi}{12}+1)\text{。}$$

(6 分)

(ii) 用 (i) 的結果，證明 $\cos\frac{\pi}{12}=\frac{\sqrt{2+\sqrt{3}}}{2}$。 (4 分)

Let $i=\sqrt{-1}$.

(a) (i) Find the two complex solutions of the equation $z^2-\sqrt{3}z+1=0$ and express them in polar form $\cos\theta+i\sin\theta$, where $-\pi<\theta\leq\pi$. (5 marks)

(ii) Using the result in (i), find all the complex solutions of the equation $z^4-\sqrt{3}z^2+1=0$ and express them in polar form $\cos\theta+i\sin\theta$, where $-\pi<\theta\leq\pi$. (5 marks)

(b) (i) Show that the polynomial $z^4-\sqrt{3}z^2+1$ has the following factorization:

$$z^4-\sqrt{3}z^2+1=\left(z^2-2z\cos\frac{\pi}{12}+1\right)\left(z^2+2z\cos\frac{\pi}{12}+1\right).$$

(6 marks)

(ii) Using the result in (i), show that $\cos\frac{\pi}{12}=\frac{\sqrt{2+\sqrt{3}}}{2}$. (4 marks)

5. (a) 因式分解行列式 $\begin{vmatrix} 1 & a & a^4 \\ 1 & b & b^4 \\ 1 & c & c^4 \end{vmatrix}$。 (7 分)

(b) (i) 求方程組 $(E)\begin{cases} x+y+z=2 \\ 3x+y-z = 4 \end{cases}$ 的通解。 (4 分)

(ii) 求 a、b 及 c 的值使得所有 (E) 的解都是方程組

$$(E')\begin{cases} x+ \quad y+ \quad z=2 \\ 3x+ \quad y- \quad z=4 \\ ax^2+by^2+cz^2=3 \end{cases}$$

的解。 (5 分)

(iii) 設在 (E') 中 $a=b=c=1$，求 (E') 的解。 (4 分)

(a) Factorize the determinant $\begin{vmatrix} 1 & a & a^4 \\ 1 & b & b^4 \\ 1 & c & c^4 \end{vmatrix}$. (7 marks)

(b) (i) Find the general solution of the system of equations $(E)\begin{cases} x+y+z=2 \\ 3x+y-z = 4 \end{cases}$. (4 marks)

(ii) Find the values of a, b and c such that all the solutions of (E) are also solutions of

$$(E')\begin{cases} x+ \quad y+ \quad z=2 \\ 3x+ \quad y- \quad z=4 \\ ax^2+by^2+cz^2=3 \end{cases}.$$ (5 marks)

(iii) Suppose in (E'), $a=b=c=1$. Find the solutions of (E'). (4 marks)

参考答案：

1. (a) 由 $|EG|=\sqrt{|EH|^2+|HG|^2}=\sqrt{2}$，得 $\angle ECG=\tan^{-1}\frac{|EG|}{|CG|}=\tan^{-1}\sqrt{2}$。

   同理，$\angle ECD=\angle ECB=\tan^{-1}\sqrt{2}$，故 $\angle MCB=\angle MCD=\angle MCG$。

   又 $|BC|=|DC|=|GC|$ 及 MC 是公共邊，得 ΔCMG、ΔCMB 和 ΔCMD 是全等三角形。

   (b) 已知 $CM\perp MG$，由 (a) 得 $CM\perp MD$，因此 $CM\perp$ 平面 MGD。

   由 (a) 得 $CM\perp MB$，因此，點 B 是在平面 MGD 內。

   故點 B、D、G 和 M 是共面的。

   明顯，點 B、D 和 G 並非共線，故平面 BDG 即平面 MGD。

   因此，$CM\perp$ 平面 BDG，且點 M 是在平面 BDG 內。

   (c) 由 (a) 得 $|MB|=|MD|=|MG|$，故點 M 是過點 B、D 和 G 的圓的圓心。

   由 (b) 得 $EM\perp$ 平面 BDG，故 $|EM|$ 是圓錐的高。圓錐的體積是 $\frac{\pi}{3}|MG|^2|EM|$。

   由 (a) 的解中，得 $\frac{|MG|}{|MC|}=\tan\angle MCG=\tan\angle ECG=\sqrt{2}$。

   又因 $|MC|^2+|MG|^2=1$，故可得 $|MC|=\frac{\sqrt{3}}{3}$ 及 $|MG|=\frac{\sqrt{6}}{3}$。

   由 $|EC|=\sqrt{|EH|^2+|HG|^2+|GC|^2}=\sqrt{3}$，得 $|EM|=|EC|-|MC|=\frac{2\sqrt{3}}{3}$。

   圓錐的體積是 $\frac{4\sqrt{3}\pi}{27}$。

2. (a)(i) $f'(x)=3x^2-2x-1$，$f''(x)=6x-2$。

   (ii) $f'(x)=0\Leftrightarrow(3x+1)(x-1)=0\Leftrightarrow x=-\frac{1}{3}$ 或 $x=1$。

   當 $-2<x<-\frac{1}{3}$ 或 $1<x<2$，$f'(x)>0$，故 $f(x)$是遞增的。

   當 $-\frac{1}{3}<x<1$，$f'(x)<0$，故 $f(x)$ 是遞減的。

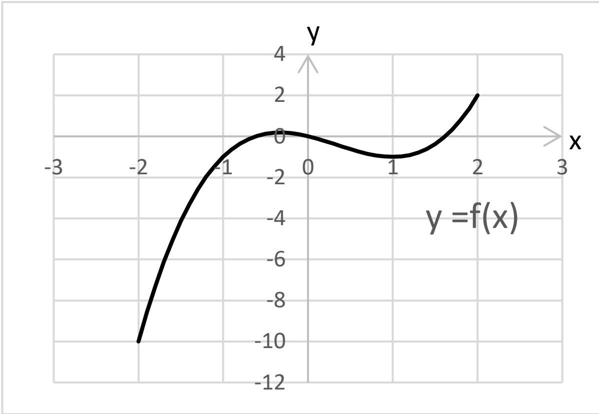
   因此，$f(-\frac{1}{3})=\frac{5}{27}$ 是一局部極大值，$f(1)=-1$ 是一局部極小值。

   (iii) $f''(x)=0\Leftrightarrow x=\frac{1}{3}$。

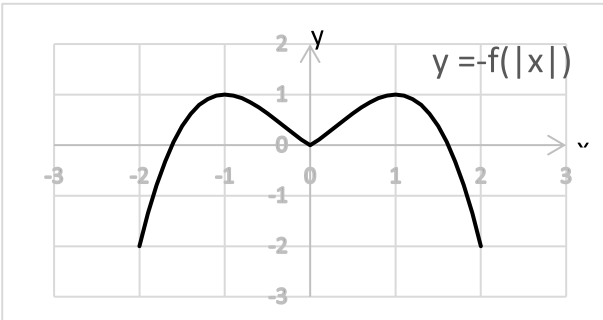
   當 $-2<x<\frac{1}{3}$，$f''(x)<0$。當 $\frac{1}{3}<x<2$，$f''(x)>0$。

   因此，$\left(\frac{1}{3},-\frac{11}{27}\right)$ 是曲線 $y=f(x)$ 的拐點。

(iv)



(v)



(b) 解 $\begin{cases} y = x^3 - x^2 - x \\ y = x \end{cases}$，得 $x = -1$ 或 $x = 0$ 或 $x = 2$。

所包圍的兩區域的總面積是

$$\begin{aligned} & \int_{-1}^{0} (x^3 - x^2 - x) - x\,dx + \int_{0}^{2} x - (x^3 - x^2 - x)\,dx \\ &= \int_{-1}^{0} x^3 - x^2 - 2x\,dx + \int_{0}^{2} 2x + x^2 - x^3\,dx \\ &= \left[\frac{x^4}{4} - \frac{x^3}{3} - x^2\right]_{-1}^{0} + \left[x^2 + \frac{x^3}{3} - \frac{x^4}{4}\right]_{0}^{2} \\ &= \frac{5}{12} + \frac{8}{3} \\ &= \frac{37}{12}\text{。} \end{aligned}$$

3. (a) 因 $a^2 = 9 - 4 = 5$，得 $a = \sqrt{5}$。圓 C 為 $x^2 + y^2 = 5$。

由 $\begin{cases} \frac{x^2}{9} + \frac{y^2}{4} = 1 \\ x^2 + y^2 = 5 \end{cases}$，得 $5x^2 = 9$，因此，$x = \pm\frac{3\sqrt{5}}{5}$。

因點 Q 是在第一象限，故 Q 的座標是 $(\frac{3\sqrt{5}}{5}, \frac{4\sqrt{5}}{5})$。

(b) 直線 QF_2的斜率是 $\frac{\frac{4\sqrt{5}}{5}-0}{\frac{3\sqrt{5}}{5}-\sqrt{5}}=-2$，直線 QF_2的方程是 $y=-2x+2\sqrt{5}$。

由 $\begin{cases} \frac{x^2}{9}+\frac{y^2}{4}=1 \\ y=-2x+2\sqrt{5} \end{cases}$，得 $5x^2-9\sqrt{5}x+18=0$，其解為 $x=\frac{6\sqrt{5}}{5}$ 或 $x=\frac{3\sqrt{5}}{5}$。

因此得點 M 是 $(\frac{6\sqrt{5}}{5},-\frac{2\sqrt{5}}{5})$。

(c) 點 M、Q 和 F_2 是共線的，直線 MQ 的斜率是 $m_{MQ}=-2$。

直線 MF_1 的斜率是 $m_{MF_1}=\frac{0-(-\frac{2\sqrt{5}}{5})}{-\sqrt{5}-\frac{6\sqrt{5}}{5}}=-\frac{2}{11}$，故

$$\tan\angle QMF_1=\left|\frac{m_{MQ}-m_{MF_1}}{1+m_{MQ}m_{MF_1}}\right|=\frac{4}{3}\text{。}$$

(d) 設 N 為直線 F_1M 上一點，使得 $QN\perp F_1M$，則所求距離為 $|NQ|$。

由 $\tan\angle QMF_1=\frac{4}{3}$，可得 $\sin\angle QMF_1=\frac{4}{5}$。

另外，$|MQ|=\sqrt{(\frac{6\sqrt{5}}{5}-\frac{3\sqrt{5}}{5})^2+(-\frac{2\sqrt{5}}{5}-\frac{4\sqrt{5}}{5})^2}=3$。

因此，$|NQ|=|MQ|\sin\angle QMF_1=\frac{12}{5}$。

4. (a) (i) 方程的解是 $\frac{\sqrt{3}\pm i}{2}$，它們的極式是 $\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}$ 和 $\cos(-\frac{\pi}{6})+i\sin(-\frac{\pi}{6})$。

(ii) 由 (i) 可知 $z^2=\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}$ 或 $z^2=\cos(-\frac{\pi}{6})+i\sin(-\frac{\pi}{6})$。

若 $z^2=\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}$，

$z=\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}$ 或 $z=\cos\left(-\frac{11\pi}{12}\right)+i\sin\left(-\frac{11\pi}{12}\right)$。

若 $z^2=\cos(-\frac{\pi}{6})+i\sin(-\frac{\pi}{6})$，

$z=\cos\left(-\frac{\pi}{12}\right)+i\sin\left(-\frac{\pi}{12}\right)$ 或 $z=\cos\left(\frac{11\pi}{12}\right)+i\sin\left(\frac{11\pi}{12}\right)$。

(b) (i) 由 (a) 可知

$$\begin{aligned}z^4-\sqrt{3}z^2+1=&\left[z-(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12})\right]\left[z-(\cos(-\frac{11\pi}{12})+i\sin(-\frac{11\pi}{12}))\right]\\&\times\left[z-(\cos(-\frac{\pi}{12})+i\sin(-\frac{\pi}{12}))\right]\left[z-(\cos\frac{11\pi}{12}+i\sin\frac{11\pi}{12})\right]\text{。}\end{aligned}$$

從

$$\begin{aligned}&\left[z-(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12})\right]\left[z-(\cos(-\frac{\pi}{12})+i\sin(-\frac{\pi}{12}))\right]\\=&\left[z-(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12})\right]\left[z-(\cos(\frac{\pi}{12})-i\sin(\frac{\pi}{12}))\right]\\=&z^2-2z\cos\frac{\pi}{12}+1\end{aligned}$$

及

$$\begin{aligned}&\left[z-(\cos(-\frac{11\pi}{12})+i\sin(-\frac{11\pi}{12}))\right]\left[z-(\cos\frac{11\pi}{12}+i\sin\frac{11\pi}{12})\right]\\=&\left[z-(\cos\frac{11\pi}{12}-i\sin\frac{11\pi}{12})\right]\left[z-(\cos\frac{11\pi}{12}+i\sin\frac{11\pi}{12})\right]\end{aligned}$$

$= z^2 - 2z\cos\frac{11\pi}{12} + 1$
$= z^2 + 2z\cos\frac{\pi}{12} + 1$
可得到結果。

(ii) 比較在 (b)(i) 的方程中 z^2 的系數，得 $-\sqrt{3} = 1 - 4\cos^2\frac{\pi}{12} + 1$。

因此有 $4\cos^2\frac{\pi}{12} = 2 + \sqrt{3}$，故 $\cos\frac{\pi}{12} = \frac{\sqrt{2+\sqrt{3}}}{2}$ $(\because \cos\frac{\pi}{12} > 0)$。

5. (a)

$$\begin{vmatrix} 1 & a & a^4 \\ 1 & b & b^4 \\ 1 & c & c^4 \end{vmatrix} = \begin{vmatrix} 1 & a & a^4 \\ 0 & b-a & b^4-a^4 \\ 0 & c-a & c^4-a^4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & a^4 \\ 0 & b-a & (b^2+a^2)(b+a)(b-a) \\ 0 & c-a & (c^2+a^2)(c+a)(c-a) \end{vmatrix}$$

$$= (b-a)(c-a)\begin{vmatrix} 1 & a & a^4 \\ 0 & 1 & (b^2+a^2)(b+a) \\ 0 & 1 & (c^2+a^2)(c+a) \end{vmatrix}$$

$$= (b-a)(c-a)[(c^2+a^2)(c+a) - (b^2+a^2)(b+a)]$$
$$= (b-a)(c-a)[c^3 - b^3 + (c-b)a^2 + a(c^2-b^2)]$$
$$= (b-a)(c-a)[(c-b)(c^2+cb+b^2) + (c-b)a^2 + a(c+b)(c-b)]$$
$$= (b-a)(c-a)(c-b)(a^2+b^2+c^2+ab+ac+bc)$$

(b)(i) $x = 1 + t$，$y = 1 - 2t$， $z = t$， $t \in \mathbb{R}$。

(ii) 把 (b)(i) 的解代入 $ax^2 + by^2 + cz^2 = 3$ 中，

得 $(a+4b+c)t^2 + (2a-4b)t + (a+b) = 3$。

此方程對所有 $t \in \mathbb{R}$ 成立，得 $\begin{cases} a+4b+c = 0 \\ 2a-4b = 0 \\ a+b = 3 \end{cases}$。

解此方程組，得 $a = 2$，$b = 1$，$c = -6$。

(iii) 把 (b)(i) 的解代入 $x^2 + y^2 + z^2 = 3$ 中，得 $6t^2 - 2t - 1 = 0$。

此方程的解為 $t = \frac{1\pm\sqrt{7}}{6}$。

因此，$(x,y,z) = (\frac{7+\sqrt{7}}{6}, \frac{4-2\sqrt{7}}{6}, \frac{1+\sqrt{7}}{6})$ 或 $(\frac{7-\sqrt{7}}{6}, \frac{4+2\sqrt{7}}{6}, \frac{1-\sqrt{7}}{6})$。

Suggested Answers:

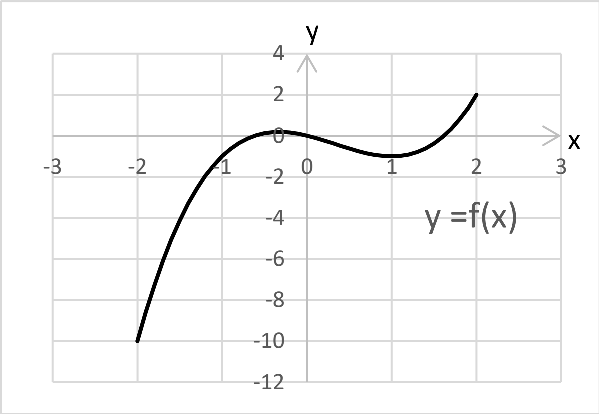
1. (a) As $|EG| = \sqrt{|EH|^2 + |HG|^2} = \sqrt{2}$, we have $\angle ECG = \tan^{-1}\frac{|EG|}{|CG|} = \tan^{-1}\sqrt{2}$.
Similarly, $\angle ECD = \angle ECB = \tan^{-1}\sqrt{2}$. Thus, $\angle MCB = \angle MCD = \angle MCG$.
Also, $|BC| = |DC| = |GC|$ and MC is a common side. Hence, ΔCMG, ΔCMB and ΔCMD are congruent triangles.

(b) Given $CM \perp MG$. From (a), we have $CM \perp MD$. Hence, $CM \perp$ plane MGD.
From (a), we have $CM \perp MB$. So, point B is in the plane MGD.
Hence, points B, D, G and M are co-planar.
Obviously, points B, D and G are not collinear and so the plane BDG is the plane MGD.
Hence, $CM \perp$ plane BDG and M is a point in the plane BDG.

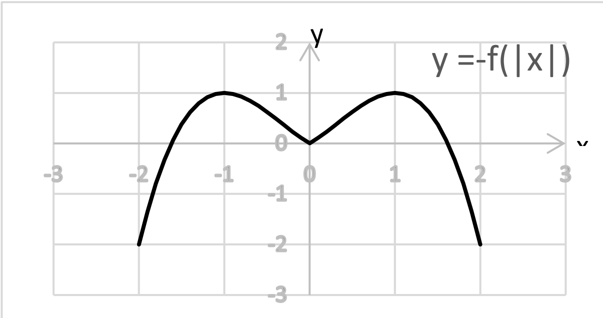
(c) From (a), we have $|MB| = |MD| = |MG|$.
Hence, M is the center of the circle passing through points B, D and G.
From (b), we have $EM \perp$ plane BDG, and so $|EM|$ is the height of the cone.
The volume of the cone is $\frac{\pi}{3}|MG|^2|EM|$.
From the solution of (a), we get $\frac{|MG|}{|MC|} = \tan\angle MCG = \tan\angle ECG = \sqrt{2}$.
With $|MC|^2 + |MG|^2 = 1$, we get $|MC| = \frac{\sqrt{3}}{3}$ and $|MG| = \frac{\sqrt{6}}{3}$.
As $|EC| = \sqrt{|EH|^2 + |HG|^2 + |GC|^2} = \sqrt{3},\ |EM| = |EC| - |MC| = \frac{2\sqrt{3}}{3}$.
The volume of the cone is $\frac{4\sqrt{3}\pi}{27}$.

2. (a)(i) $f'(x) = 3x^2 - 2x - 1$, $f''(x) = 6x - 2$.
(ii) $f'(x) = 0 \Leftrightarrow (3x+1)(x-1) = 0 \Leftrightarrow x = -\frac{1}{3}$ or $x = 1$.
When $-2 < x < -\frac{1}{3}$ or $1 < x < 2$, $f'(x) > 0$. So, $f(x)$ is increasing.
When $-\frac{1}{3} < x < 1$, $f'(x) < 0$. So, $f(x)$ is decreasing.
Hence, $f(-\frac{1}{3}) = \frac{5}{27}$ is a local maximum value; $f(1) = -1$ is a local minimum value.
(iii) $f''(x) = 0 \Leftrightarrow x = \frac{1}{3}$.
When $-2 < x < \frac{1}{3}$, $f''(x) < 0$. When $\frac{1}{3} < x < 2$, $f''(x) > 0$.
Hence, $\left(\frac{1}{3}, -\frac{11}{27}\right)$ is an inflection point of the curve $y = f(x)$.

(iv)



(v)



(b) Solving $\begin{cases} y = x^3 - x^2 - x \\ y = x \end{cases}$, we get $x = -1$ or $x = 0$ or $x = 2$.

The total area of the two regions bounded is

$$\begin{aligned} &\int_{-1}^{0} (x^3 - x^2 - x) - x\, dx + \int_{0}^{2} x - (x^3 - x^2 - x)\, dx \\ &= \int_{-1}^{0} x^3 - x^2 - 2x\, dx + \int_{0}^{2} 2x + x^2 - x^3\, dx \\ &= \left[\frac{x^4}{4} - \frac{x^3}{3} - x^2\right]_{-1}^{0} + \left[x^2 + \frac{x^3}{3} - \frac{x^4}{4}\right]_{0}^{2} \\ &= \frac{5}{12} + \frac{8}{3} \\ &= \frac{37}{12}. \end{aligned}$$

3. (a) As $a^2 = 9 - 4 = 5$, we get $a = \sqrt{5}$. The circle C is $x^2 + y^2 = 5$.

From $\begin{cases} \frac{x^2}{9} + \frac{y^2}{4} = 1 \\ x^2 + y^2 = 5 \end{cases}$, we get $5x^2 = 9$. So, $x = \pm\frac{3\sqrt{5}}{5}$.

As Q is in the first quadrant, the coordinates of Q are $(\frac{3\sqrt{5}}{5}, \frac{4\sqrt{5}}{5})$.

(b) The slope of line QF_2 is $\frac{\frac{4\sqrt{5}}{5}-0}{\frac{3\sqrt{5}}{5}-\sqrt{5}} = -2$. An equation of line QF_2 is $y = -2x + 2\sqrt{5}$.

From $\begin{cases} \frac{x^2}{9} + \frac{y^2}{4} = 1 \\ y = -2x + 2\sqrt{5} \end{cases}$, we get $5x^2 - 9\sqrt{5}x + 18 = 0$ and its solutions are $x = \frac{6\sqrt{5}}{5}$ or $x = \frac{3\sqrt{5}}{5}$.

Hence, point M is $(\frac{6\sqrt{5}}{5}, -\frac{2\sqrt{5}}{5})$.

(c) The points M, Q and F_2 are collinear. The slope of line MQ is $m_{MQ} = -2$.

The slope of line MF_1 is $m_{MF_1} = \frac{0-(-\frac{2\sqrt{5}}{5})}{-\sqrt{5}-\frac{6\sqrt{5}}{5}} = -\frac{2}{11}$. Hence,

$$\tan \angle QMF_1 = \left|\frac{m_{MQ} - m_{MF_1}}{1 + m_{MQ}m_{MF_1}}\right| = \frac{4}{3}.$$

(d) Let N be the point on line F_1M such that $QN \perp F_1M$. Then, the required distance is $|NQ|$.

From $\tan \angle QMF_1 = \frac{4}{3}$, we get $\sin \angle QMF_1 = \frac{4}{5}$.

Also, $|MQ| = \sqrt{(\frac{6\sqrt{5}}{5} - \frac{3\sqrt{5}}{5})^2 + (-\frac{2\sqrt{5}}{5} - \frac{4\sqrt{5}}{5})^2} = 3$.

Hence, $|NQ| = |MQ| \sin \angle QMF_1 = \frac{12}{5}$.

4. (a) (i) The solutions of the equation are $\frac{\sqrt{3}\pm i}{2}$. In polar form,

$\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$ and $\cos(-\frac{\pi}{6}) + i\sin(-\frac{\pi}{6})$.

(ii) By (i) we have $z^2 = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$ or $z^2 = \cos(-\frac{\pi}{6}) + i\sin(-\frac{\pi}{6})$.

For $z^2 = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$,

$z = \cos\frac{\pi}{12} + i\sin\frac{\pi}{12}$ or $z = \cos(-\frac{11\pi}{12}) + i\sin(-\frac{11\pi}{12})$.

For $z^2 = \cos(-\frac{\pi}{6}) + i\sin(-\frac{\pi}{6})$,

$z = \cos(-\frac{\pi}{12}) + i\sin(-\frac{\pi}{12})$ or $z = \cos(\frac{11\pi}{12}) + i\sin(\frac{11\pi}{12})$.

(b) (i) From (a), we have

$$\begin{aligned} z^4 - \sqrt{3}z^2 + 1 = &\left[z - (\cos\frac{\pi}{12} + i\sin\frac{\pi}{12})\right]\left[z - (\cos(-\frac{11\pi}{12}) + i\sin(-\frac{11\pi}{12}))\right] \\ &\times\left[z - (\cos(-\frac{\pi}{12}) + i\sin(-\frac{\pi}{12}))\right]\left[z - (\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12})\right]. \end{aligned}$$

The result follows from

$$\begin{aligned} &\left[z - (\cos\frac{\pi}{12} + i\sin\frac{\pi}{12})\right]\left[z - (\cos(-\frac{\pi}{12}) + i\sin(-\frac{\pi}{12}))\right] \\ &= \left[z - (\cos\frac{\pi}{12} + i\sin\frac{\pi}{12})\right]\left[z - (\cos(\frac{\pi}{12}) - i\sin(\frac{\pi}{12}))\right] \\ &= z^2 - 2z\cos\frac{\pi}{12} + 1 \end{aligned}$$

and

$$\begin{aligned} &\left[z - (\cos(-\frac{11\pi}{12}) + i\sin(-\frac{11\pi}{12}))\right]\left[z - (\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12})\right] \\ &= \left[z - (\cos\frac{11\pi}{12} - i\sin\frac{11\pi}{12})\right]\left[z - (\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12})\right] \\ &= z^2 - 2z\cos\frac{11\pi}{12} + 1 \end{aligned}$$

$= z^2 + 2z\cos\frac{\pi}{12} + 1.$

(ii) Comparing the coefficients of z^2 in the equation in (b)(i), we get $-\sqrt{3} = 1 - 4\cos^2\frac{\pi}{12} + 1$. So, $4\cos^2\frac{\pi}{12} = 2 + \sqrt{3}$ and hence $\cos\frac{\pi}{12} = \frac{\sqrt{2+\sqrt{3}}}{2}$ $(\because \cos\frac{\pi}{12} > 0)$.

5. (a)

$$\begin{vmatrix} 1 & a & a^4 \\ 1 & b & b^4 \\ 1 & c & c^4 \end{vmatrix} = \begin{vmatrix} 1 & a & a^4 \\ 0 & b-a & b^4-a^4 \\ 0 & c-a & c^4-a^4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & a^4 \\ 0 & b-a & (b^2+a^2)(b+a)(b-a) \\ 0 & c-a & (c^2+a^2)(c+a)(c-a) \end{vmatrix}$$

$$= (b-a)(c-a)\begin{vmatrix} 1 & a & a^4 \\ 0 & 1 & (b^2+a^2)(b+a) \\ 0 & 1 & (c^2+a^2)(c+a) \end{vmatrix}$$

$$= (b-a)(c-a)[(c^2+a^2)(c+a) - (b^2+a^2)(b+a)]$$
$$= (b-a)(c-a)[c^3 - b^3 + (c-b)a^2 + a(c^2-b^2)]$$
$$= (b-a)(c-a)[(c-b)(c^2+cb+b^2) + (c-b)a^2 + a(c+b)(c-b)]$$
$$= (b-a)(c-a)(c-b)(a^2+b^2+c^2+ab+ac+bc)$$

(b)(i) $x = 1+t,\ y = 1-2t,\ z = t,\ t \in \mathbb{R}$.

(ii) Putting the solution found in (b)(i) into $ax^2 + by^2 + cz^2 = 3$,

we have $(a+4b+c)t^2 + (2a-4b)t + (a+b) = 3$.

As this equation is satisfied by all $t \in \mathbb{R}$, we have $\begin{cases} a+4b+c = 0 \\ 2a-4b = 0 \\ a+b = 3 \end{cases}$.

Solving this system of equations, we get $a = 2, b = 1, c = -6$.

(iii) Putting the solution found in (b)(i) into $x^2 + y^2 + z^2 = 3$,

we have $6t^2 - 2t - 1 = 0$. The solutions of this equation are $t = \frac{1\pm\sqrt{7}}{6}$.

Hence, $(x, y, z) = (\frac{7+\sqrt{7}}{6}, \frac{4-2\sqrt{7}}{6}, \frac{1+\sqrt{7}}{6})$ or $(\frac{7-\sqrt{7}}{6}, \frac{4+2\sqrt{7}}{6}, \frac{1-\sqrt{7}}{6})$.